A Minimal Model for Human and Nature Interaction

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Abstract

There are widespread concerns that current trends in population and resource-use are unsustainable, but the possibilities of an overshoot and collapse remain unclear and controversial. Collapses have occurred frequently in the past five thousand years, and are often followed by centuries of economic, intellectual, and population decline. Many different natural and social phenomena have been invoked to explain specific collapses, but a general explanation remains elusive. Two important features seem to appear across societies that have collapsed: (1) Ecological Strain and (2) Economic Stratification.

In this paper, the structure of a new model and several simulated scenarios that offer significant implications are explained. The model has just four equations that describe the evolution of the populations of Elites and Commoners, Nature, and accumulated Wealth. Mechanisms leading to collapse are discussed and the measure "Carrying Capacity" is developed and defined. The model suggests that the estimation of Carrying Capacity is a practical means for early detection of a collapse. Collapse can be avoided, and population can reach a steady state at the maximum carrying capacity, if the rate of depletion of nature is reduced to a sustainable level, and if resources are distributed equitably.

1 Introduction

There are widespread concerns that current trends in population and resource-use are unsustainable, but the possibilities of an overshoot and collapse remain unclear and controversial. How real is the possibility of societal collapse? Can complex, advanced civilizations really collapse? It is common to see human history as a relentless and inevitable trend toward greater levels of social complexity, political organization, and economic specialization, with the development of more complex and capable technologies supporting ever-growing population, all sustained by the mobilization of ever-increasing quantities of material, energy and information. Yet this is not inevitable. In fact, cases where this seemingly near-universal, long-term trend has been severely disrupted by a precipitous collapse —often lasting centuries— have been quite common. A brief review of some examples of collapses suggests that the process of rise-and-collapse is actually a recurrent cycle found throughout history, making it important to establish a general explanation of this process [Tainter, 1988; Yoffee and Cowgill, 1988; Turchin and Nefedov, 2009; Chase-Dunn and Hall, 1997; Goldstein, 1988; Modelski, 1987; Meadows et al., 1972].

The Roman Empire's dramatic collapse (followed by many centuries of population decline, economic deterioration, intellectual regression and the disappearance of literacy) is well known, but it was not the first rise-and-collapse cycle in Europe. Prior to the rise of Classical Greco-Roman civilization, both the Minoan and Mycenaean Civilizations had each risen, reached very advanced levels of civilization, and then collapsed virtually completely [Morris, 2005; Redman, 1999]. The history of Mesopotamia, the very cradle of civilization, agriculture, complex society and urban life, presents a series of rise-and-declines including the Sumerians, the Akkadian, Assyrian, Babylonian, Achaemenid, Seleucid, Parthian, Sassanid, Umayyad, and Abbasid Empires [Yoffee, 1979; Redman et al., 2004]. In neighboring Egypt, this cycle also appeared repeatedly. In both Anatolia and in the Indus Valley, the very large and long-lasting Hittite and Harrapan civilizations both collapsed so completely that their very existence was unknown until modern archeology rediscovered them. Similar cycles of rise and collapse occurred repeatedly in India, most notably with the Mauryan and the Gupta Empires [Thapar, 2004; Jansen et al., 1991; Kenoyer, 1998; Edwards et al., 1971, 1973]. Chinese history is, very much like Egypt's, full of repeated cycles of rises and collapses, with each of the Zhou, Han, Tang and Song Empires followed by a very serious collapse of political authority and socioeconomic progress [Chu and Lee, 1994; Needham and Wang, 1956; Lee, 1931]. Collapses are not restricted to the "Old World". The collapse of Maya Civilization is well known and evokes widespread fascination, both because of the advanced nature of Mayan society and because of the depth of the collapse [Webster, 2002; Demerest et al., 2004]. As Jared Diamond [Diamond, 2005] puts it, it is difficult to ignore "the disappearance of between 90 and 99% of the Maya population after A.D. 800 ... and the disappearance of kings, Long Count calendars, and other complex political and cultural institutions." In the central Highlands of Mexico, a number of powerful states also rose to high levels of power and prosperity and then rapidly collapsed, Teotihuacan (the sixth largest city in the world in the 7th C) and Monte Alban being just the largest of these to experience dramatic collapse, with their populations declining to about 20-25% of their peak within just a few generations[Tainter, 1988]. We know of many other collapses, and it is likely that other collapses have also occurred in societies that were not at a sufficient level of complexity to produce written records or archeological evidence. Despite the common impression that societal collapse is rare, or even largely fictional, "The picture that emerges is of a process recurrent in history, and global in its distribution" [Tainter, 1988]. See also Yoffee and Cowgill [1988]; Goldstein [1988]; Ibn Khaldun [1958]; Kondratieff [1984]; Parsons [1991]. As Turchin and Nefedov [Turchin and Nefedov, 2009] contend, there is a great deal of support for "the hypothesis that secular cycles —demographic-social-political oscillations of a very long period (centuries long) are the rule, rather than an exception in the large agrarian states and empires".

This brings up the question of whether modern civilization is similarly susceptible. It may be reasonable to believe that modern civilization, armed with its greater technological capacity, scientific knowledge, and energy resources, will be able to survive and endure whatever crises historical societies succumbed to. But the brief overview of collapses demonstrates not only the ubiquity of the phenomenon, but also the extent to which advanced, complex and powerful societies are susceptible to collapse. The fall of the Roman Empire, and the equally —if not more— advanced Han, Mauryan and Gupta Empires, as well as so many advanced Mesopotamian Empires, are all testimony to the fact that advanced, sophisticated, complex, and creative civilizations can be both fragile and impermanent.

A large number of explanations have been proposed for each specific case of collapse, including one or more of the following: volcanoes, earthquakes, droughts, floods, changes in the courses of rivers, soil degradation (erosion, exhaustion, salinization, etc), deforestation, tribal migrations, foreign invasions, changes in technology (such as the introduction of ironworking), changes in the methods or weapons of warfare (such as the introduction of horse cavalry, armored infantry or long swords), changes in trade patterns, depletion of particular mineral resources (e.g. silver mines), cultural decline and social decadence, popular uprisings, and civil wars. However, these explanations are specific to each particular case of collapse rather than general. Moreover, even for the specific case where the explanation applies, the society in question usually had already experienced the phenomenon identified as the cause without collapsing. For example, the Minoan society had repeatedly experienced earthquakes that destroyed palaces, and they simply rebuilt them more splendidly than before. Indeed, many societies experience droughts, floods, volcanoes, soil erosion, and deforestation with no major social disruption. The same applies to migrations, invasions and civil wars. The Roman, Han, Assyrian, and Mauryan Empires were, for centuries, completely hegemonic, successfully defeating the neighboring "barbarian" peoples who eventually did overrun them. So external military pressure alone hardly constitutes an explanation for their collapses. With both natural disasters and external threats, identifying a specific cause compels one to ask, "yes, but why did this particular instance of this factor produce the collapse?" Other processes must be involved, and, in fact, the political, economic, ecological, and technological conditions under which civilizations have collapsed have varied widely. Individual collapses may have involved an array of specific factors, with particular triggers, but a general explanation remains elusive. Individual explanations may seem appropriate in their particular case, but the very universal nature of the phenomenon implies a mechanism that is not specific to a particular time period of human history, nor a particular culture, technology, or natural disaster [Tainter, 1988; Yoffee and Cowgill, 1988; Turchin, 2003].

In this paper we attempt to model collapse mathematically in a more general way. We propose a simple model, not intended to describe actual individual cases, but rather to provide a general framework that allows carrying out "thought experiments" for the phenomenon of collapse and to test changes that would avoid it. Two important features seem to appear across societies that have collapsed: (1) the stretching of resources due to the strain placed on the ecological carrying capacity [Ponting, 1991; Redman, 1999; Redman et al., 2004; Kammen, 1994; Postan, 1966; Ladurie, 1987; Abel, 1980; Catton, 1980; Wood, 1998], and (2) the economic stratification of society into Elites and Masses (or "Commoners") [Brenner, 1985; Parsons, 1991; Turchin, 2005, 2006; Turchin and Nefedov, 2009; Diamond, 2005; Goldstone, 1991; Ibn Khaldun, 1958]. In many of these historical cases, we have direct evidence of Ecological Strain and Economic Stratification playing a central role in the character or in the process of the collapse [Diamond, 2005; Goldstone, 1991; Culbert, 1973; Lentz, 2000; Mitchell, 1990]. For this reason, our model includes these two features. Although, like the Brander-Taylor (BT) model [Brander and Taylor, 1998], HANDY is based on the classical predator-prey model, the inclusion of two societal classes introduces a much richer set of dynamical solutions, including cycles of societal and ecological collapse, as well as the possibility of smoothly reaching equilibrium (the ecological carrying capacity). We use Carrying Capacity in its biological definition, as the population level that the resources of a particular environment can maintain over the long term [Catton, 1980; Daly and Farley, 2003; Cohen, 1995].

In this paper, we call these environmental resources "Nature".

The paper is organized as follows: section 2 gives a brief review of the Predator-Prey model, section 3 includes the mathematical description of HANDY, section 4 is a theoretical analysis of the model equilibrium and possible solutions, section 5 presents examples of scenarios within three distinct types of societies, section 6 presents an overall discussion of the scenarios from section 5, and section 7 includes a short summary of the paper and a discussion of future work.

2 Predator-Prey Model

The predator-prey model was the original inspiration behind HANDY. This system of equations was derived independently by two mathematicians, Alfred Lotka and Vitto Volterra, in the early 20th century [Lotka, 1925; Volterra, 1926]. This model describes the dynamics of competition between two species, say, wolves and rabbits. The governing system of equations is

$$\begin{cases} \dot{x} &= (ay)x - bx \\ \dot{y} &= cy - (dx)y \end{cases} \tag{1}$$

In the above system, x represents the predator (wolf) population; y represents the prey (rabbit) population; a determines the predator's birth rate, i.e., the faster growth of wolf population due to availability of rabbits; b is the predator's death rate; c is the prey's birth rate; d determines the predation rate, i.e., the rate at which rabbits are hunted by wolves.

The predator and prey populations show periodic, out-of-phase variations about the equilibrium values

$$\begin{cases} x_e &= c/d \\ y_e &= b/a \end{cases} \tag{2}$$

A typical solution of the predator-prey system can be seen below:

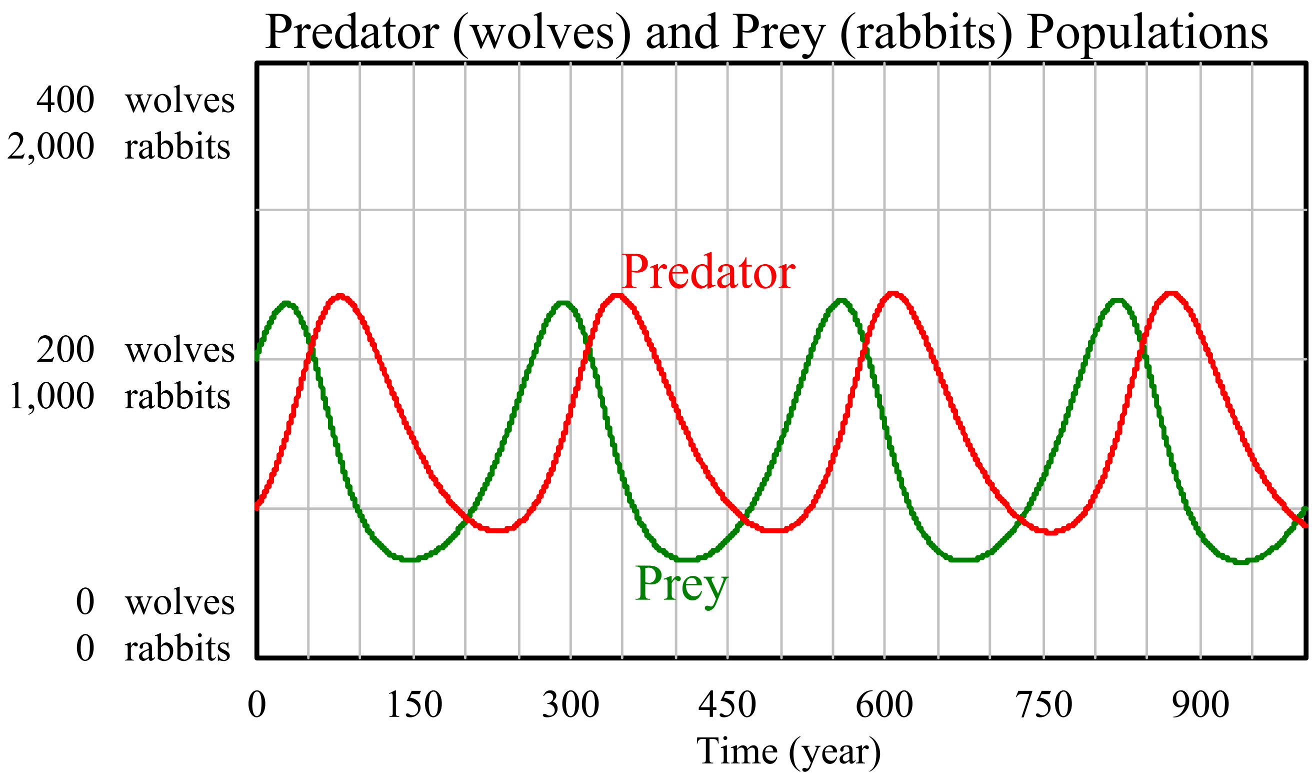

Figure 1: A typical solution of the predator-prey system

This typical solution can be obtained by running the system with the following parameter values and initial conditions:

$$\begin{cases} a = 3.0 \times 10^{-5} \text{ (rabbits.years)}^{-1} & b = 2.0 \times 10^{-2} \text{ years}^{-1} \\ c = 3.0 \times 10^{-2} \text{ years}^{-1} & d = 2.0 \times 10^{-4} \text{ (wolves.years)}^{-1} \\ x(0) = 1.0 \times 10^{+2} \text{ wolves} & y(0) = 1.0 \times 10^{+3} \text{ rabbits} \end{cases} \quad (3)$$

Note consistency of the units on the left and right hand sides of (1) and (2). Predator population is measured in units of *wolves*, Prey population is measured in units of *rabbits*, and Time is measured in units of *years*.

3 HANDY

As indicated above, Human And Nature DYnamics (HANDY) was originally built based on the predator-prey model. We can think of the human population as the "predator", while nature (the natural resources of the surrounding environment) can be taken as the "prey", depleted by humans. Based on the long history of collapse of civilizations discussed in the introduction, we separated the population into "Elites" and "Commoners", and introduced a variable for accumulated wealth. For an analysis of this two-class structure of modern society, see Drăgulescu and Yakovenko [2001]; Banerjee and Yakovenko [2010]. We have also added a different dimension of predation whereby Elites "prey" on the production of wealth by Commoners. As a result, HANDY consists of just four prediction equations: two for the two classes of population, Elites and Commoners, denoted by x_E and x_C, respectively, one for the natural resources or Nature, y, and one for the accumulated Wealth, w, referred to hereafter as "Wealth". This minimal set of four equation seems to capture essential features of the human-nature interaction and is capable of producing major potential scenarios of collapse or transition to steady state.

A similar model of population and renewable resource dynamics based on the predator-prey model was developed in the pioneering work of Brander and Taylor [1998], demonstrating that reasonable parameter values can produce cyclical "feast and famine" patterns of population and resources. Their model showed that a system with a slow-growing resource base will exhibit overshooting and collapse, whereas a more rapidly growing resource base will produce an adjustment of population and resources toward equilibrium values. They then applied this model to the historical case of Easter Island, finding that the model provides a plausible explanation of the population dynamics known about Easter Island from the archeological and scientific record. They thus argue that the Polynesian cases where population did collapse were due to smaller maximum resource bases (which they call carrying capacity) that grew more slowly, whereas those cases which did not experience such a collapse were due to having a larger resource base (i.e., a larger carrying capacity). They then speculate whether their model might be consistent with other historical cases of collapse, such as the ancient Mesopotamian and Maya civilizations or modern Rwanda.

However, the Brander-Taylor approach only models Population and Nature and does not include a central component of these historical cases: economic stratification and the accumulation of wealth. Brander and Taylor recognize that their model is simple, and that application to more complex scenarios may require further development of the structure of the model. We have found that including economic stratification, in the form of the introduction of Elites and Commoners, as well as accumulated Wealth, results in a much richer variety of solutions, which may have a wider application across different types of societies. Thus while the Brander-Taylor model has only two equations, HANDY has four equations to predict the evolution of the rich and poor populations (Elites and Commoners), Nature, and accumulated Wealth. (We examine other differences in section 6.4 of the paper.) The HANDY equations are given by:

$$\begin{cases} \dot{x}_C &= \beta_C x_C - \alpha_C x_C \\ \dot{x}_E &= \beta_E x_E - \alpha_E x_E \\ \dot{y} &= \gamma y(\lambda - y) - \delta x_C y \\ \dot{w} &= \delta x_C y - C_C - C_E \end{cases} \tag{4}$$

3.1 Model Description

The total population is divided between the two variables, x_C and x_E, representing the population of masses and of elites. The population grows through a birth rate β and decreases through a death rate α. β is assumed to be constant for both Elites and Commoners but α depends on Wealth as explained below.

Natural resources exist in three forms: nonrenewable stocks (fossil fuels, mineral deposits, etc), renewable stocks (forests, soils, aquifers), and flows (wind, solar radiation, rivers). In future versions of HANDY, we plan to disaggregate Nature into these three different forms, but for simplification in this version, we have adopted a single formulation intended to represent an amalgamation of the three forms. Thus, he equation for Nature includes a regeneration term, $\gamma y(\lambda - y)$, and a depletion term, $-\delta x_C y$. The regeneration term has been written in the form of a logistic equation, with a regeneration factor, γ, and exponential regrowth for low values of y, and saturation when y approaches λ, Nature's capacity —maximum size of Nature in absence of depletion [Brander and Taylor, 1998]. As a result, the maximum rate of regeneration takes place when $y = \lambda/2$. Production is understood according to standard Ecological Economics formulations as involving both inputs from, and outputs to, Nature (i.e., depletion of natural resources and pollution of natural sinks) [Daly and Farley, 2003; Daly, 1996]. This initial version of HANDY models the Depletion side of the equation as if it includes the reduction in Nature due to Pollution. Future versions will differentiate Depletion from Pollution. The depletion term includes a rate of depletion per worker, δ, and is proportional to both Nature and the number of workers. However, the economic activity of Elites is modeled to represent executive, management, and supervisory functions, but not engagement in the direct extraction of resources, which is done by Commoners. Thus, only Commoners produce.

Technological change can raise the efficiency of resource use, but it also tends to raise both per capita resource consumption and the scale of resource of extraction, such that, absent policy effects, the increases in consumption often compensate for the increased efficiency of resource use. These are associated with the phenomena referred to as the Jevon's Paradox, and the "Rebound Effect" [Polimeni et al., 2008; Greening et al., 2000]. For example, an increase in vehicle fuel-efficiency technology tends to enable increased per capita vehicle miles driven, heavier cars, and higher average speeds, which then negate the gains from the increased fuel-efficiency. The extent of these effects varies, but in this initial model, we assume that the effects of these trends tend to cancel each other out. In future versions, the rates of these trends could be adjusted in either direction.

Finally, there is an equation for accumulated Wealth, which increases with production, $\delta x_C y$, and decreases with the consumption of the Elites and the Commoners, C_C and C_E, respectively. The consumption of the Commoners (as long as there is enough wealth to pay them) is sx_C, a subsistence salary per capita, s, multiplied by the working population. The Elites pay themselves a salary κ times larger, so that the consumption of the Elites is $\kappa s x_E$. However, once the wealth becomes too small to pay for this consumption, i.e., when $w < w_{th}$, the payment is reduced and eventually stopped, and famine takes place, with a much higher rate of death. κ is meant to represent here the factors that determine the division of the output of the total production of society between elites and masses, such as the balance of class power between elites and masses, and the capacity of each group to organize and pursue their economic interests. In this initial version of the model, we hold that balance (κ) constant in each scenario, but we expect to develop it further in later versions, so that it can be endogenously determined by other factors in the model.

C_C and C_E, the consumption rates for the Commoner and the Elite respectively, are given by the following equations:

$$\begin{cases} C_C = \min\left(1, \dfrac{w}{w_{th}}\right) s x_C \\ C_E = \min\left(1, \dfrac{w}{w_{th}}\right) \kappa s x_E \end{cases} \tag{5}$$

Wealth threshold, w_{th}, is a threshold value for wealth below which famine starts. It depends on the "minimum required consumption per capita", ρ:

$$w_{th} = \rho x_C + \kappa \rho x_E. \tag{6}$$

Even when Commoners start experiencing famine, i.e., when $w \leq w_{th}$, the Elites continue consuming unequally as indicated by the factor κ in the second term on the right hand side of (6). A graphical representation of the consumption rates are given in the figure below.

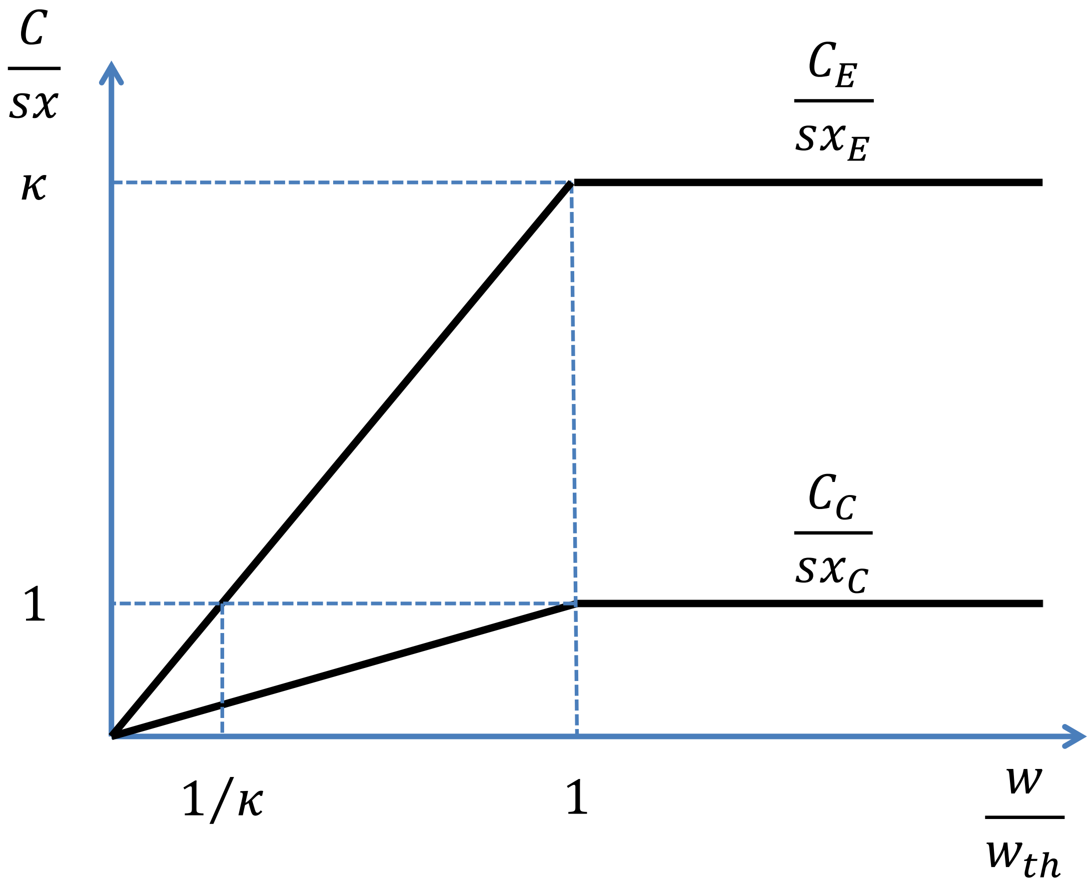

Figure 2: Consumption rates for Elites and Commoners as a function of Wealth. Famine starts when $\frac{C}{sx} \leq 1$. Therefore, Commoners start experiencing famine when $\frac{w}{w_{th}} \leq 1$, while Elites do not experience famine until $\frac{w}{w_{th}} \leq \frac{1}{\kappa}$.

The death rates for the Commoner and the Elite, α_C and α_E, are functions of consumption rates:

$$\begin{cases} \alpha_C = \alpha_m + \max\left(0, 1 - \frac{C_C}{sx_C}\right)(\alpha_M - \alpha_m) \\ \alpha_E = \alpha_m + \max\left(0, 1 - \frac{C_E}{sx_E}\right)(\alpha_M - \alpha_m) \end{cases} \tag{7}$$

The death rates vary between a normal (healthy) value, α_m, observed when there is enough food for subsistence, and a maximum (famine) value, α_M that prevails when the accumulated wealth has been used up and the population starves. The death rates α_C and α_E can be expressed equivalently in terms of $\frac{w}{w_{th}}$, a graphical representation of which is given below.

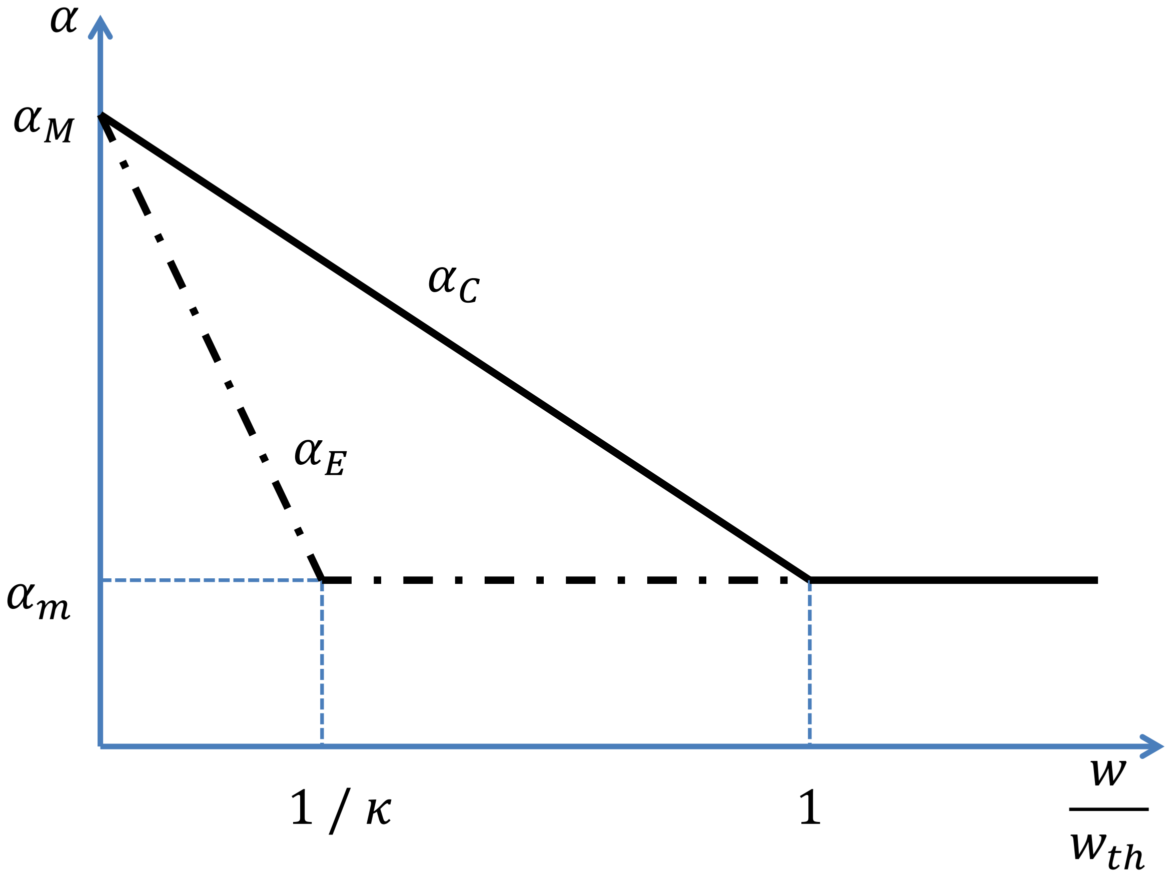

Figure 3: Death rates for Elites and Commoners as a function of Wealth. Elites experience famine with a delay due to their unequal access to Wealth.

3.2 A Note on Units and Dimensions

There are three dimensions for quantities in HANDY:

1. Population (either Commoner or Elite), in units of *people*, also shown as *ppl*.
2. Nature/Wealth, in units of *"eco-Dollars"/"Eco-Dollars"*. (Capitalization is only used to distinguish different scales for Nature and Wealth on the subsequent graphs.)
3. Time, in units of *years*.

The structure of model requires Nature and Wealth to be measured with the same units, therefore we created the unit eco-dollar to serve this purpose. Other parameters and functions in the model carry units that are compatible with the abovementioned dimensions following (4). For example, Carrying Capacity, χ, and Maximum Carrying Capacity, χ_M, defined in section 4.1, are both expressed in units of people (ppl).

4 Equilibrium Values and Carrying Capacity

We can use the model to find a sustainable equilibrium and maximum carrying capacity in different types of societies. In order for population to reach an equilibrium, we must have $\alpha_m \leq \beta_E \leq \beta_C \leq \alpha_M$. We define a dimensionless parameter, η:

$$\eta = \frac{\alpha_M - \beta_C}{\alpha_M - \alpha_m} \tag{8}$$

Since we assume $\alpha_m \leq \beta_C \leq \alpha_M$, $0 \leq \eta \leq 1$.

4.1 Equilibrium when $x_E = 0$ (No Elites)

Assuming $x_E \equiv 0$, we can find the equilibrium values of the system:

$$\begin{cases} x_{C,e} &= \dfrac{\gamma}{\delta}\left(\lambda - \eta\dfrac{s}{\delta}\right) \\ y_e &= \eta\dfrac{s}{\delta} \\ w_e &= \eta\rho x_{C,e} \end{cases} \tag{9}$$

We define χ, the Carrying Capacity for the population, to be equal to $x_{C,e}$ in (9), i.e., the equilibrium value of the population in the absence of Elites:

$$\chi = \frac{\gamma}{\delta}\left(\lambda - \frac{s}{\delta}\eta\right) \tag{10}$$

Carrying capacity can be maximized if Nature's regeneration rate is maximal, i.e., if $y_e = \frac{\lambda}{2}$. This requires δ to be set equal to its optimal value, δ_*. From the second equation in (9), it can be seen that δ_* is given by:

$$\delta_* = \frac{2\eta s}{\lambda} \tag{11}$$

The Maximum Carrying Capacity, χ_M, is thus given by:

$$\chi_M = \frac{\gamma}{\delta_*}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2 \tag{12}$$

4.2 Equilibrium when $x_E \geq 0$ and $\kappa = 1$ (No Inequality)

If we set $\kappa \equiv 1$ and $\beta_E \equiv \beta_C \equiv \beta$, we can reach an equilibrium state for which $x_E \geq 0$. This case models an equitable society of "Workers" and "Non-Workers". We need a dimensionless free parameter φ that sets the initial ratio of the Non-Workers to Workers:

$$\varphi = \frac{x_E(0)}{x_C(0)} \tag{13}$$

The equilibrium values of the system can then be expressed as follows:

$$\begin{cases} x_{C,e} &= \dfrac{\gamma}{\delta}\left(\lambda - \eta\dfrac{s}{\delta}(1+\varphi)\right) \\ x_{E,e} &= \varphi x_{C,e} \\ y_e &= \eta\dfrac{s}{\delta}(1+\varphi) \\ w_e &= \eta\rho(1+\varphi)x_{C,e} \end{cases} \tag{14}$$

The total population $x_e = x_{C,e} + x_{E,e}$ can still be maximized by choosing δ appropriately:

$$\delta_{**} = \frac{2\eta s}{\lambda}(1+\varphi) \tag{15}$$

This δ_{**} is larger than the optimal depletion factor given by (11). The difference arises because Workers have to produce more than they need just for themselves in order to support Non-Workers. For this choice of δ, total population is given by:

$$x_{e,M} = (1+\varphi)\frac{\gamma}{\delta_{**}}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2 \tag{16}$$

As can be seen from (16), maximum total population in equilibrium is independent of φ and conforms to the maximum carrying capacity given above by (12).

4.3 Equilibrium when $x_E \geq 0$ and $\kappa \geq 1$ (Unequal Society)

It is possible to attain equilibrium in an unequal society if we can satisfy the following condition:

$$\frac{\alpha_M - \beta_E}{\kappa(\alpha_M - \alpha_m)} = \frac{\alpha_M - \beta_C}{\alpha_M - \alpha_m} = \eta. \tag{17}$$

The general condition $\alpha_m \leq \beta_E \leq \beta_C \leq \alpha_M$ must hold in all cases for an equilibrium to be feasible.

The equilibrium values in this general case can be expressed as follows:

$$\begin{cases} x_{C,e} &= \dfrac{\gamma}{\delta}\left(\lambda - \eta\dfrac{s}{\delta}(1+\kappa\psi)\right) \\ x_{E,e} &= \psi x_{C,e} \\ y_e &= \eta\dfrac{s}{\delta}(1+\kappa\psi) \\ w_e &= \eta\rho(1+\kappa\psi)x_{C,e} \end{cases} \tag{18}$$

The free parameter, ψ, is the equilibrium ratio $x_{E,e}/x_{C,e}$, apparent from the second equation in (18). As opposed to φ, ψ cannot be easily related to the initial conditions; rather, it can be determined from the result of a simulation.

Again, the total population $x_e = x_{C,e} + x_{E,e}$ can be maximized by choosing δ appropriately:

$$\delta_{***} = \frac{2\eta s}{\lambda}(1+\kappa\psi) \tag{19}$$

This required depletion rate δ_{***} can be even larger than the optimal δ given by (15) depending upon the values of κ and ψ. In the presence of inequality, the maximum total population is no longer independent of κ and ψ and is smaller than the maximum carrying capacity given by equations (12) and (16):

$$x_{e,M} = (1+\psi)\frac{\gamma}{\delta_{***}}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2\left(\frac{1+\psi}{1+\kappa\psi}\right) \tag{20}$$

5 Scenarios

We will discuss three sets of scenarios:

1. Egalitarian society (No-Elites): Scenarios in which $x_E = 0$.
2. Equitable society (with Workers and Non-Workers): Scenarios in which $x_E \geq 0$ but $\kappa \equiv 1$.
3. Unequal society (with Elites and Commoners): Scenarios in which $x_E \geq 0$ and $\kappa \geq 1$.

For all of these scenarios, we start the model with the following parameter values and initial conditions, unless otherwise stated:

$$\begin{cases} \alpha_m = 1.0\times10^{-2} & \alpha_M = 7.0\times10^{-2} \\ \beta_C = 3.0\times10^{-2} & \beta_E = 3.0\times10^{-2} \\ \gamma = 1.0\times10^{-2} & \lambda = 1.0\times10^{+2} \\ s = 5.0\times10^{-4} & \rho = 5.0\times10^{-3} \\ x_C(0) = 1.0\times10^{+2} & \\ y(0) = \lambda & w(0) = 0 \end{cases} \tag{21}$$

As indicated above, the values of κ and $x_E(0)$ determine the type of society. Within each type of society, we obtain different scenarios by varying the depletion factor δ.

In this section, we will show that HANDY is capable of modeling three distinct types of societies by changing κ and $x_E(0)$. By controlling δ, each society can attain a sustainable equilibrium. Appropriate choice of δ can make this equilibrium optimal, i.e., with maximum total population. Increasing δ above its optimal value makes the approach toward equilibrium oscillatory. Such an equilibrium is suboptimal, and Carrying Capacity is below its Maximum value, χ_M. It is also possible to reach a suboptimal equilibrium by making δ lower than its optimal value. However, in the latter case, the approach toward equilibrium would be a soft landing rather than oscillatory.

When δ is increased even further, the society goes into cycles of prosperity and collapse. Increasing δ beyond a certain point will result in a Type-II collapse (full), examples of which are presented in sections 5.1.4, 5.2.4, and 5.3.2.

It is important to understand the inter-relation of the depletion factor, δ, and the Carrying Capacity, χ. The further δ is taken away from its optimal value, the further χ moves away from its maximum value, χ_M. An equilibrium can be reached if and only if χ is not too far away from χ_M, which means δ cannot be too far away from its optimal value, given by equations (11), (15), and (19) in the three types of societies under consideration. Note that in all of the scenario outputs presented below (for the three types of societies under consideration), Carrying Capacity (χ) and the Maximum Carrying Capacity (χ_M) are calculated from their defining equations (10) and (12), respectively.

5.1 Egalitarian Society (No-Elites)

In the four following scenarios, κ does not play any role since we set $x_E \equiv 0$. We start the depletion rate from $\delta = \delta_*$, the optimal equilibrium value that maximizes the Carrying Capacity, and increase it slowly to get additional scenarios. The horizontal red line in the graphs for the four scenarios of this section represents the *zero* population of Elites.

5.1.1 Soft Landing to Equilibrium when $x_E = 0$

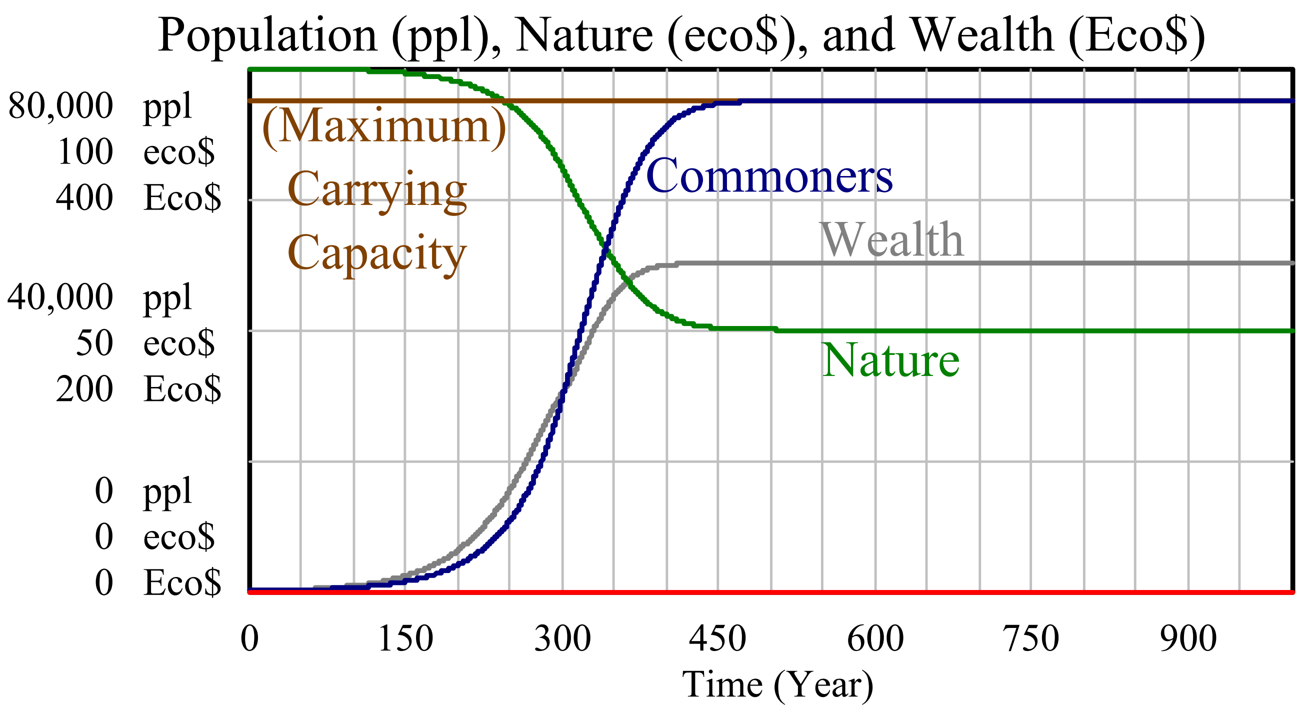

Figure 4: Soft landing to the optimal equilibrium when Elite population (marked in red) equals zero.

In this case, $\delta = \delta_* = 6.67 \times 10^{-6}$. Therefore, the carrying capacity, χ, is at its maximum level, χ_M. Notice that Nature also settles to $y_e = \lambda/2$, which is the value that results in the maximum regeneration rate. This maximal regeneration can in turn support a maximum sustainable production and population.

If we set $\delta < \delta_*$, we still see a soft landing to the carrying capacity, χ. However, χ would be at a lower level than χ_M because a sub-optimal δ cannot result in the maximum regenration of nature, which is a necessity if we want to have the *maximum* sustainable population. The advantage of a sub-optimal δ is a higher equilibrium level (compared to $\lambda/2$) for Nature.

It should be understood that choosing δ too small makes any equilibrium impossible simply because Commoners cannot even feed themselves and their population quickly collapses even though Nature stays at its maximum capacity, λ. This is not a usual case as the urge for survival guarantees humans extract their basic needs from nature, especially when natural resources are abundant.

5.1.2 Oscillatory Approach to Equilibrium when Elite population (marked in red) equals zero.

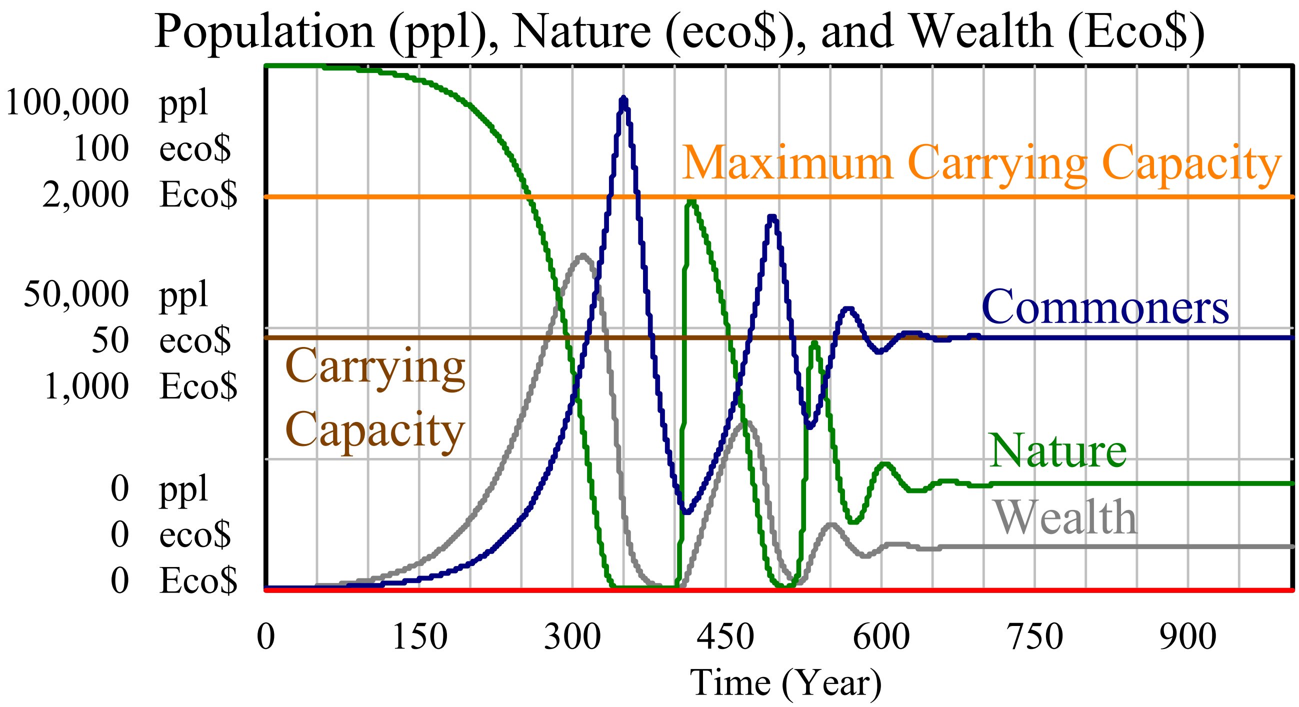

Figure 5: Oscillatory approach to equilibrium when Elite population (marked in red) equals zero.

In this scenario, δ is increased to $\delta = 2.5\delta_* = 1.67 \times 10^{-5}$. As can be seen from figure 5, the carrying capacity, χ, is lower than its maximum value, χ_M. Population initially overshoots the carrying capacity, then oscillates and eventually converges to it since the amount of overshoot is not too large, just about the order of χ. Note that at the time the (total) population overshoots the Carrying Capacity, the Wealth also reaches a maximum and starts to decline.

5.1.3 Cycles of Prosperity and Collapse when $x_E = 0$

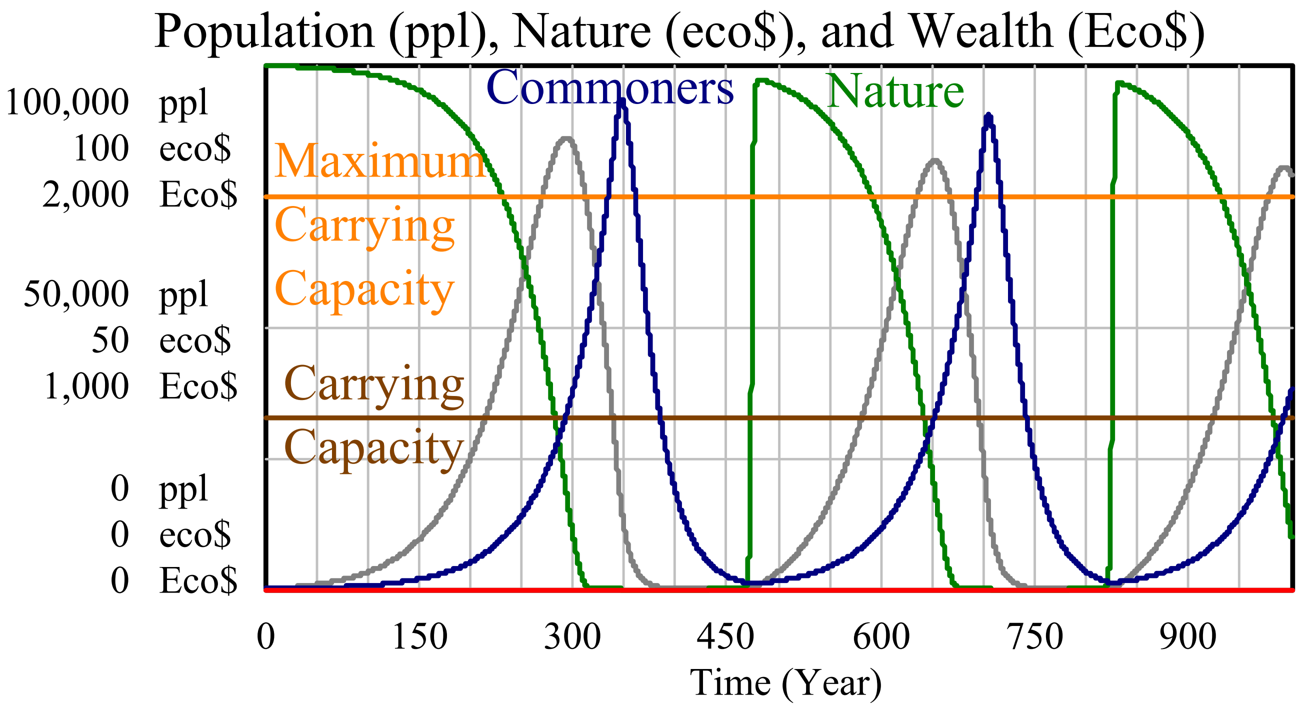

Figure 6: Cycles of prosperity and collapse when Elite population (marked in red) equals zero.

In this scenario, δ is increased to $\delta = 4\delta_* = 2.67 \times 10^{-5}$. As can be seen, Population, Nature and Wealth all collapse to a very small value. However, after depletion becomes small due to very low number of workers, Nature gets a chance to grow back close to its capacity, λ. The regrowth of Nature kicks off another cycle of prosperity which ends with another collapse. Simulation results show that these cycles repeat themselves indefinitely.

5.1.4 Type-II (Full) Collapse when $x_E = 0$

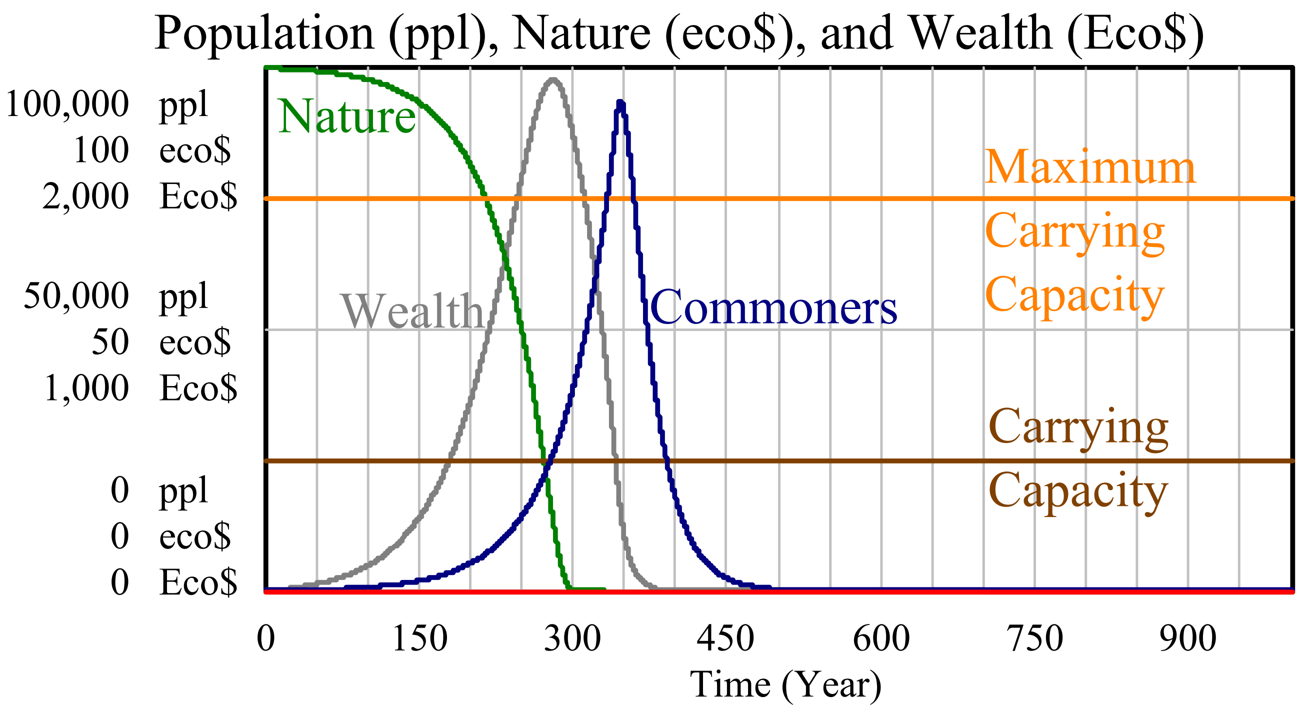

Figure 7: Type-II (full) collapse when Elite population (marked in red) equals zero. All the state variables collapse to *zero* in this scenario due to over-depletion.

In this scenario, δ is increased further to $\delta = 5.5\delta_* = 3.67E-5$. The overshoot is so large that forces Population, Nature and Wealth into a full collapse, after which there is no recovery. This is a generic type of collapse that can happen for any type of society due to over-depletion. See sections 5.2.4 and 5.3.2 for examples of a Type-II collapse in equitable and unequal societies, respectively.

5.2 Equitable society (with Workers and Non-Workers)

We take the parameter values and the initial conditions to be the same as (21), except that this time we set $x_E(0) = 25$ ($\varphi = 0.25$) and $\kappa = 1$. We start with the optimal production per capita $\delta = \delta_{**}$ (see (15)) and will gradually increase it in order to get the additional scenarios in this subsection. Notice that in these cases, x_C describes the Working Population, while x_E describes the Non-Working Population. Everybody consumes at the same level, since we set $\kappa = 1$, i.e., we assume there is no inequality in consumption level for Workers and Non-Workers.

5.2.1 No-Inequality: Soft Landing to Optimal Equilibrium

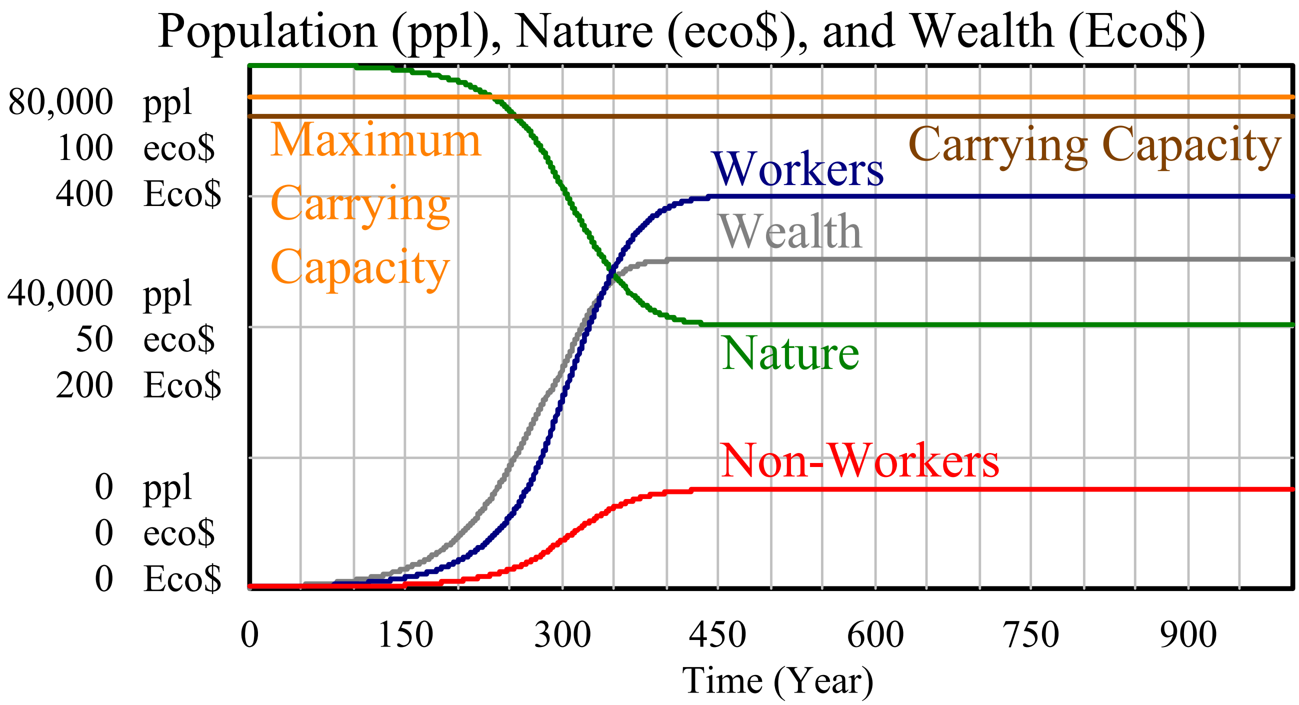

Figure 8: Equilibrium in the presence of both Workers and Non-Workers can be attained with slow growth and equitable salaries.

In this case, $\delta = \delta_{**} = 8.33 \times 10^{-6}$. Notice that this is larger than the optimal value in the absence of Non-Workers $\delta_{*} = 6.67 \times 10^{-6}$ even though all the other parameters are identical to those in section 5.1.1. This difference arises because $x_E \neq 0$, which in turn forces the workers to produce extra in order to support the Non-Workers. Now, $\chi < \chi_M$ because $\delta = \delta_{**} \neq \delta_{*}$. However, by setting $\delta = \delta_{**}$, the optimal value of δ in the presence of Non-Workers, the total population, $x_C + x_E$ still reaches the maximum Carrying Capacity χ_M, the same as in section 5.1. See equation (16) and section 4.2 for a mathematical description.

Similar comments as in section 5.1.1 apply here when we choose a sub-optimal δ.

5.2.2 No-Inequality: Oscillatory Approach to Equilibrium

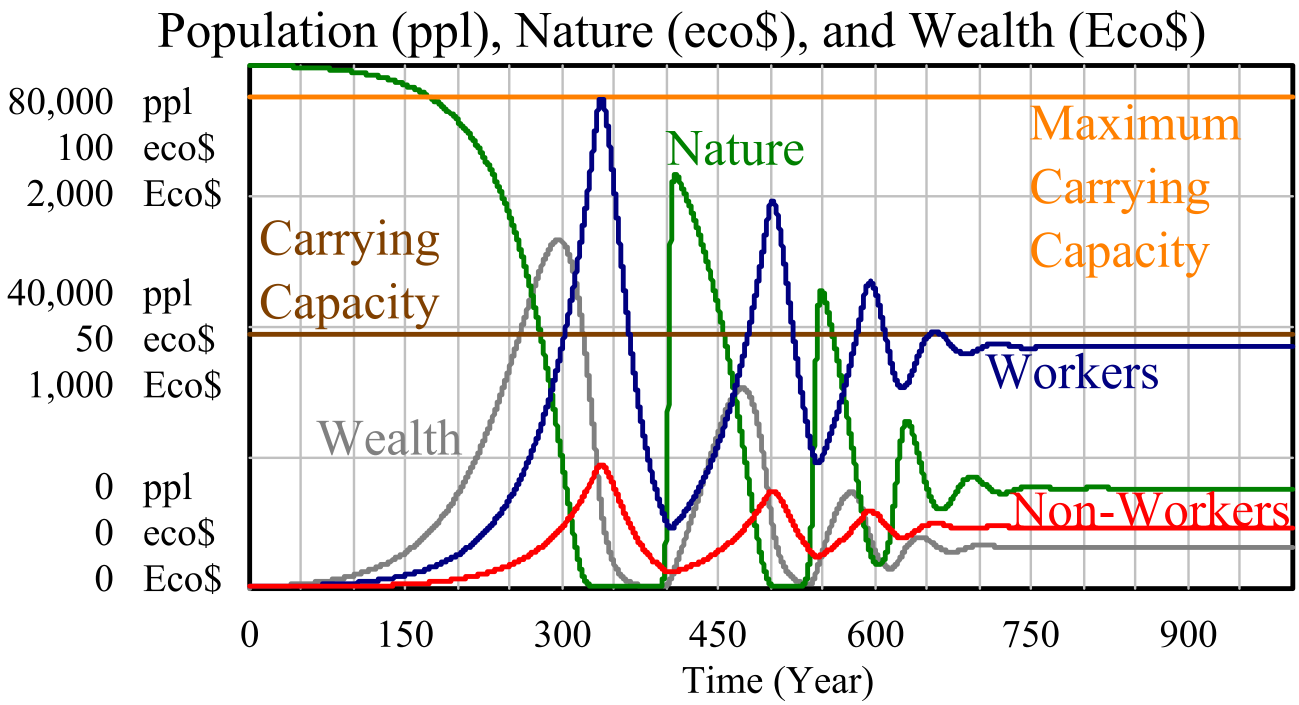

Figure 9: Oscillatory approach to equilibrium in the presence of both Workers and Non-Workers is possible when the overshoot is not too large.

In this case, $\delta = 2.64\delta_{**} = 2.20 \times 10^{-5}$. The total population is equal to the actual Carrying Capacity (smaller than the maximum Carrying Capacity).

5.2.3 No-Inequality: Cycles of Prosperity, Overshoot and Collapse

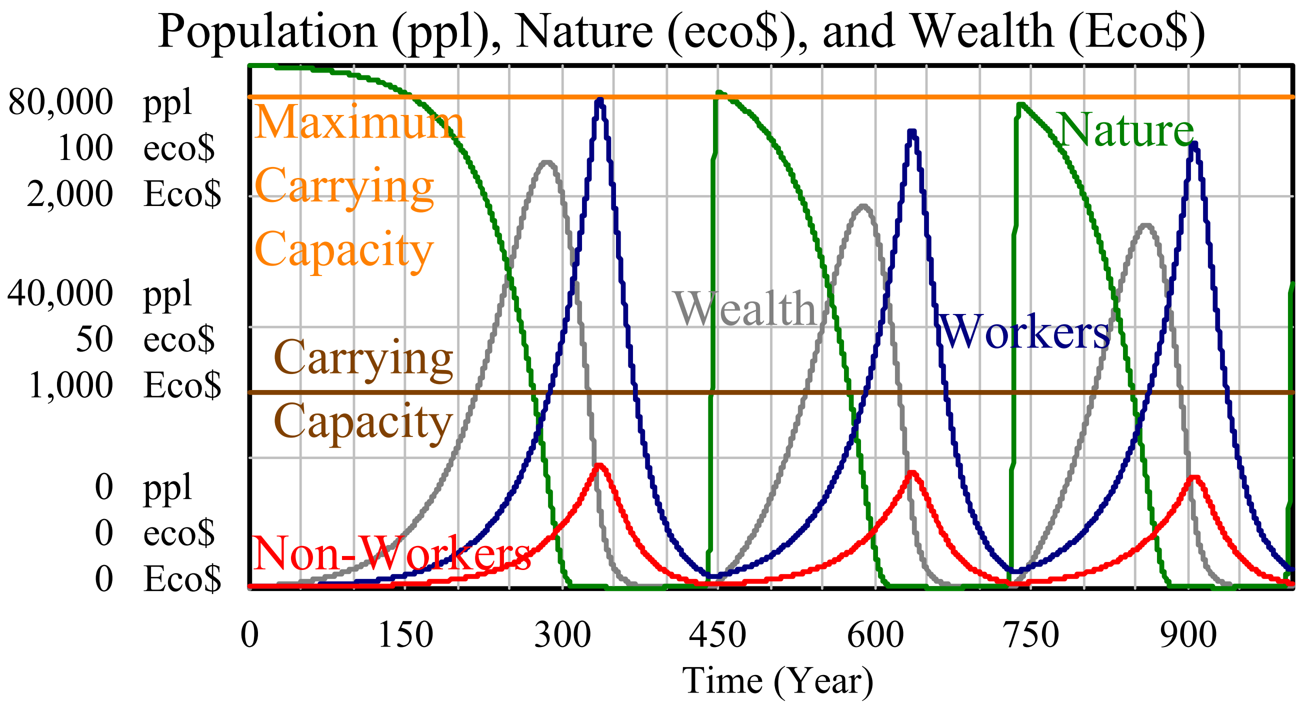

Figure 10: Cycles of prosperity, overshoot and collapse in the presence of Workers and Non-Workers

In this case, $\delta = 3.46\delta_{**} = 3.00 \times 10^{-5}$. The result is similar to figure 6 presented in section 5.1.3. As before, the time at which the total population overshoots the actual Carrying Capacity is indicated by the fact that Wealth starts to decrease. Partial collapses that occur after each cycle of prosperity are of Type-II, even though they are followed by another cycle of growth. See section 5.3.2 for a discussion of a Type-II collapse.

5.2.4 No-Inequality: Full Collapse

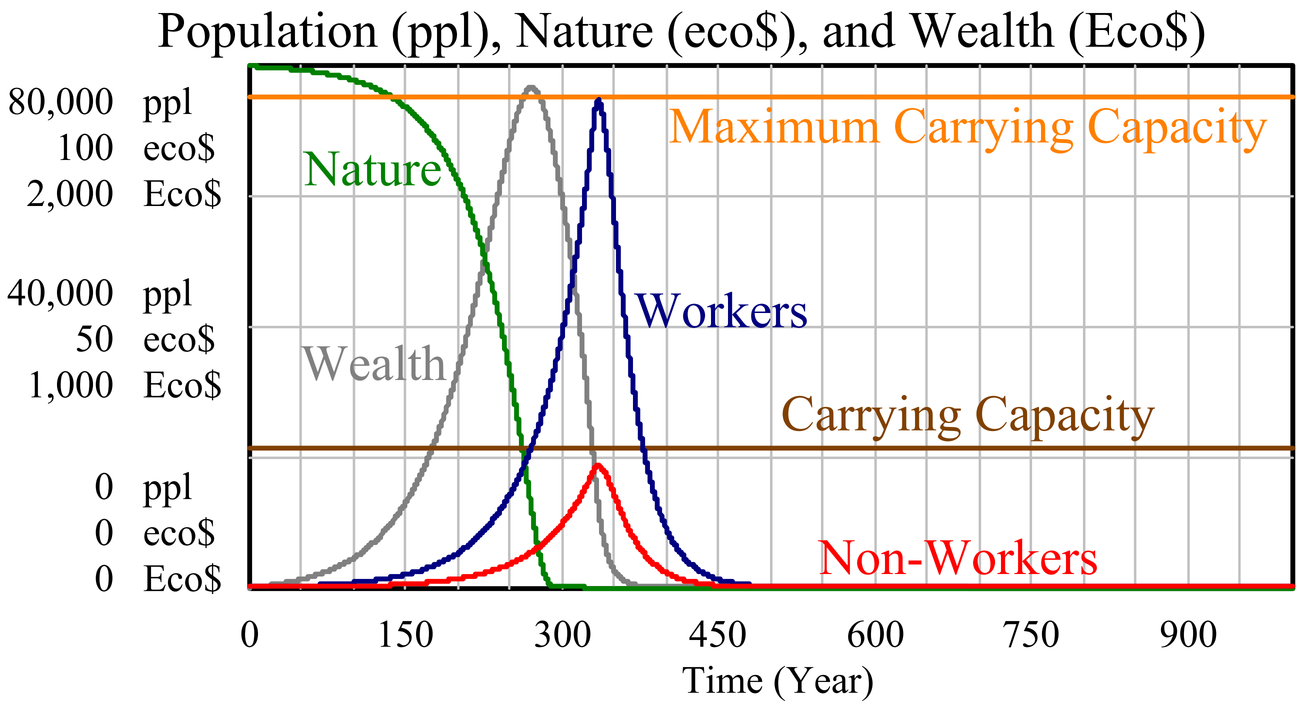

Figure 11: Type-II (full) collapse happens after a period of very fast growth.

In this case, $\delta = 5\delta_{**} = 4.33 \times 10^{-5}$. Once again, we can see how a full collapse of Population, Nature, and Wealth can occur due to over-depletion of natural resources as a result of high depletion per capita.

5.2.5 No-Inequality: Preventing a Full Collapse by Decreasing *Average* Depletion per Capita

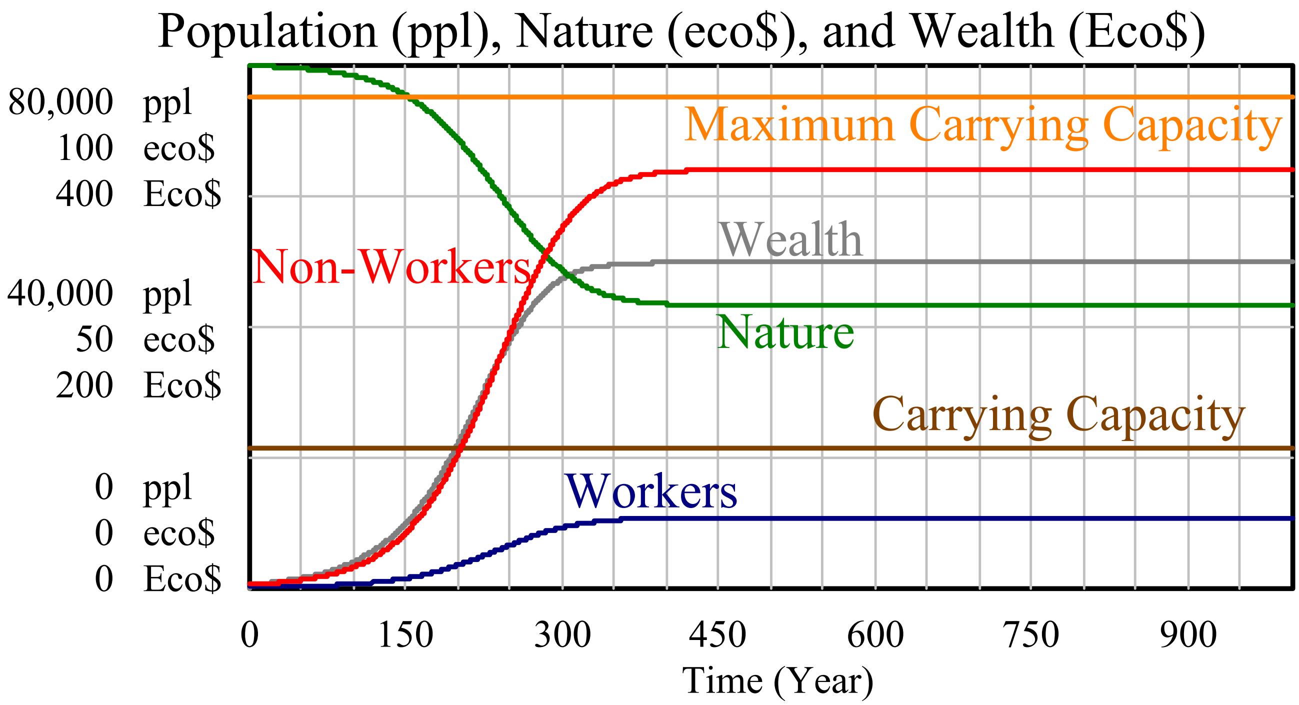

Figure 12: The full collapse that happened in the previous scenario, figure 11, can be prevented by reducing the *average* depletion per capita. This can be achieved by either increasing the ratio of the Non-Working population or decreasing the average workload per worker, i.e., decreasing total work hours per week.

This case is similar to the previous case (see section 5.2.4), except that we raised the ratio of Non-Workers to Workers, φ, from 0.25 to 6. This corresponds to changing $x_E(0)$ from 25 to 600, while keeping $x_C(0) = 100$. By increasing the ratio of non-workers to workers, a sustainable equilibrium can be reached due to lower *average* depletion per capita. This could also be interpreted as modeling a reduction in the average workload per worker.

5.3 Unequal Society (with Elites and Commoners): $x_E \geq 0$ and $\kappa \geq 1$

In our example of an unequal society, the Elites consume $\kappa \sim 10 - 100$ times more than the Commoners. Their population, plotted in red, is multiplied by κ to represent their equivalent impact because of their higher consumption. That is why we use the label "*Equivalent* Elites" on the graphs in this section, 5.3.

In the first two cases, we will discuss two distinct, but generic types of collapse in an unequal society. In these two scenarios, $\kappa = 100$. Then we will show possibility of reaching an equilibrium by reducing κ to 10 and adjusting the birth rates β_E and β_C independently. These two $\kappa = 10$ scenarios show that in order to reach a sustainable equilibrium in an unequal society, it is necessary to have policies that limit inequality and control birth rates.

5.3.1 Unequal Society: Type-I Collapse (Recovery of Nature)

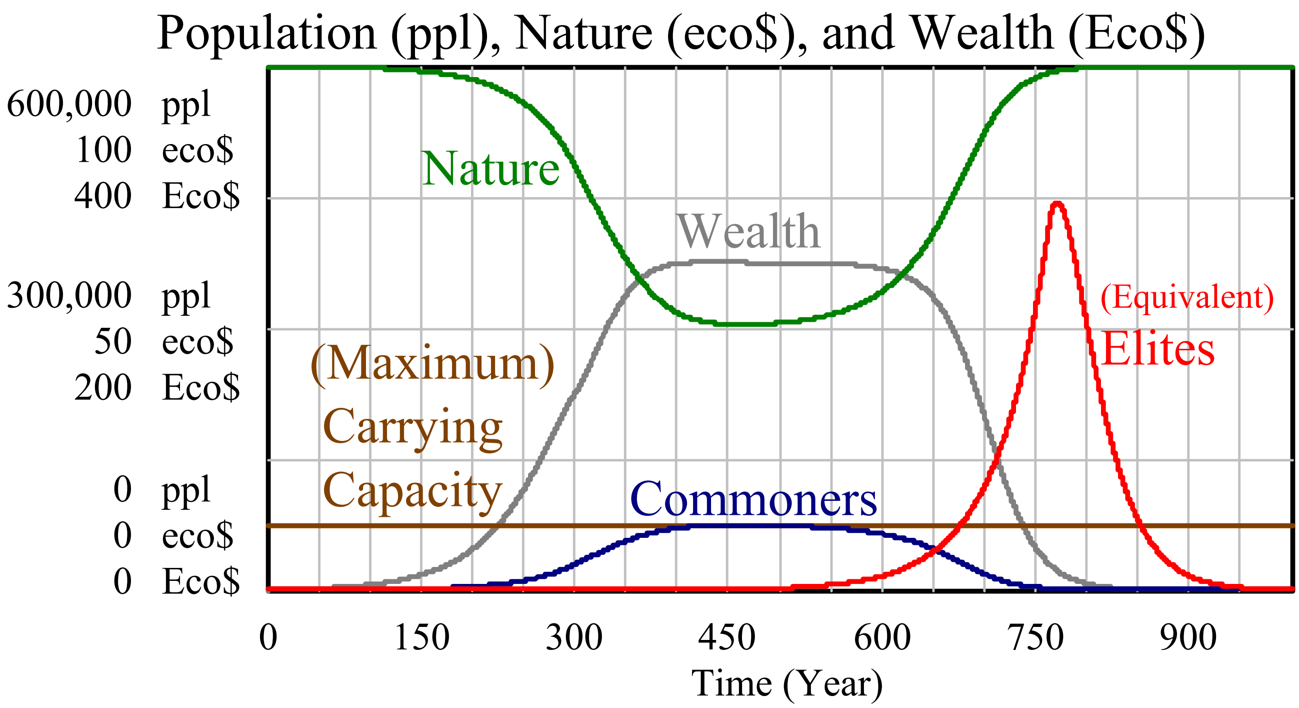

Figure 13: Population collapse following an apparent equilibrium due to a small initial Elite population when $\kappa = 100$.

This scenario is precisely the same as the equilibrium without Elites case presented in 5.1.1 except that here we set $x_E(0) = 1.0 \times 10^{-3}$. This is indeed a very small initial seed of Elites. The two scenarios look pretty much the same up until about $t = 500$ years after the starting time of the simulation. The Elite population starts growing significantly only after $t = 500$, hence depleting the Wealth and causing the system to collapse. Under this scenario, the system collapses due to the scarcity of workers even though natural resources are still abundant, but because the depletion rate is optimal, it takes more than 400 years after the Wealth reaches a maximum for the society to collapse. In this example, Commoners die out first and Elites disappear later.

This scenario is one example of a Type-I collapse in which both Population and Wealth collapse but Nature recovers (to its maximum capacity, λ, in the absence of depletion). Scarcity of workers is the initial cause of a Type-I collapse, as opposed to scarcity of Nature for a Type-II collapse. Recovery of Nature distinguishes a Type-I from a Type-II collapse.

5.3.2 Unequal Society: Type-II Collapse (Full Collapse)

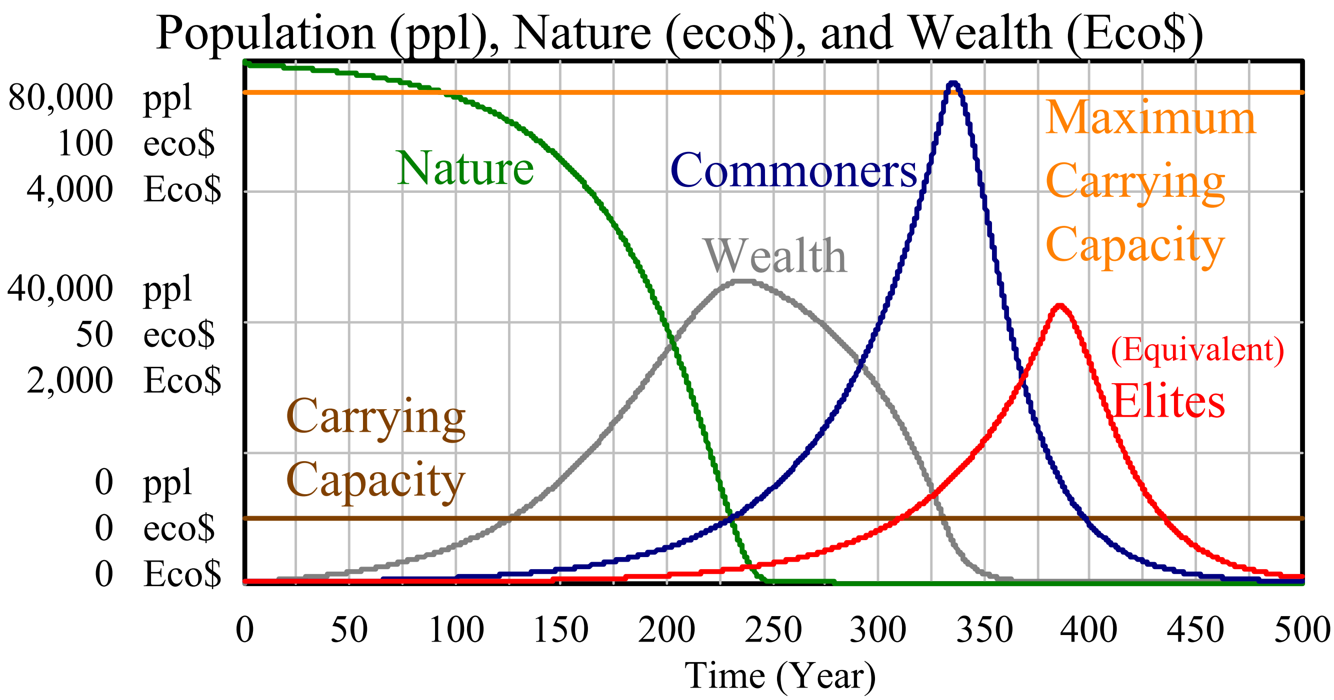

Figure 14: A fast full collapse due to both over-depletion and inequality ($\kappa = 100$).

This typical scenario for a full collapse is the result of running the model with the parameter values and initial conditions given by (21). Examples of a Type-II (full) collapse in the egalitarian and equitable societies are discussed in sections 5.1.4 and 5.2.4.

We set a small initial seed of $x_E(0) = 0.20$, $\kappa = 100$, and a large depletion $\delta = 1.0 \times 10^{-4}$, so that both the depletion $\delta = 15\delta_*$ and the inequality coefficient $\kappa = 100$ are very large. This combination results in a full collapse of the system with no recovery. The Wealth starts declining as soon as the Commoner's population goes beyond its carrying capacity, and then the full collapse takes only about 250 additional years. The declining Wealth causes the fall of the Commoner's population (workers) with a time lag. The fast reduction in the number of workers combined with scarcity of natural resources causes the Wealth to decline even faster than before. As a result, the Elites —who could initially survive the famine due to their unequal access to consumable goods ($\kappa = 100$)— eventually also die of hunger. Note that because both depletion and inequality are large, the collapse takes place faster and at a much lower level of population than in the previous case (see section 5.3.1) with a depletion rate of $\delta = \delta_*$.

5.3.3 Unequal Society: Soft Landing to Optimal Equilibrium

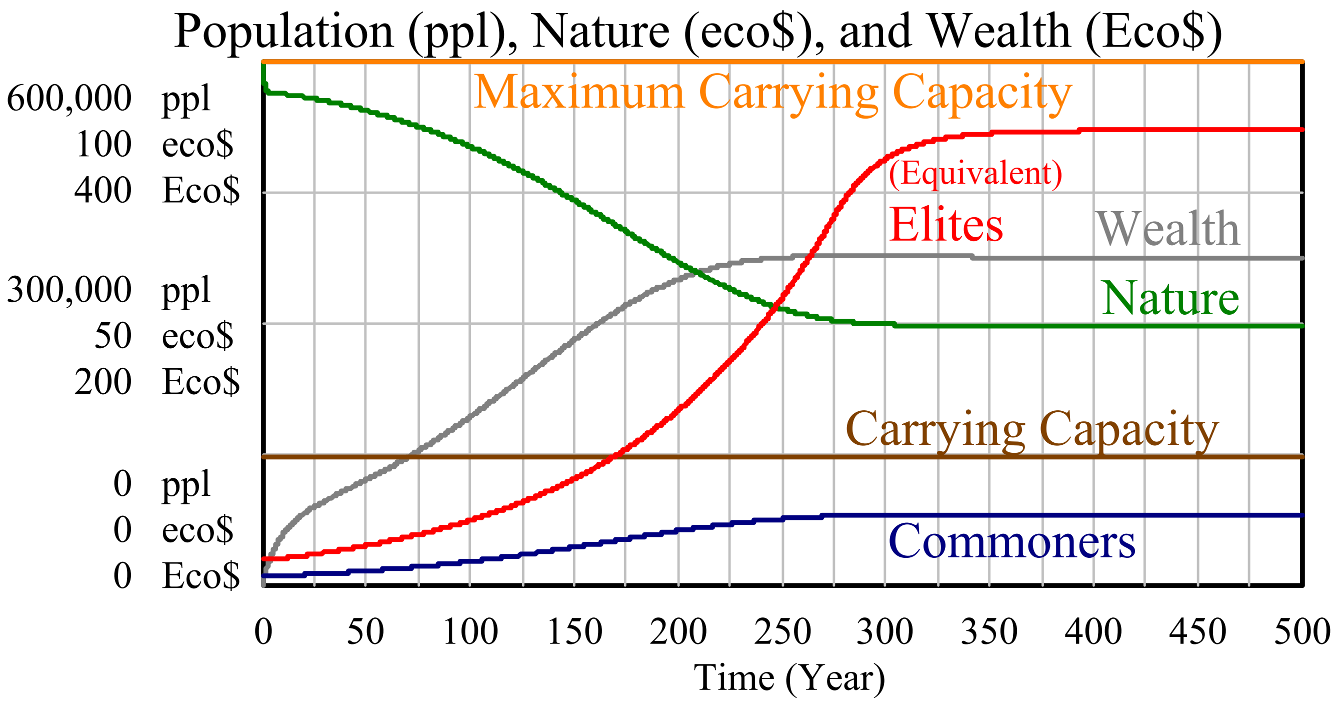

Figure 15: With moderate inequality ($\kappa = 10$), it is possible to attain an optimal equilibrium by controlling the birth rates.

The following parameter values and initial values can produce the current scenario (the rest are exactly the same as (21)):

$$\begin{cases} \beta_C = 3.0 \times 10^{-2} & \beta_E = 3.0 \times 10^{-2} \\ x_C(0) = 1.0 \times 10^{+4} & x_E(0) = 3.0 \times 10^{+3} \\ \kappa = 10 & \delta = 6.35 \times 10^{-6} \end{cases} \quad (22)$$

The value for δ used in this scenario is δ_{***} given by equation (19). It must be remembered that $\psi = 0.65$ is not a parameter that we can choose. However, it can be read from the result of the simulation since it is the equilibrium ratio of the Elite to Commoner population. See the second equation in (18). On the other hand, $\eta = \frac{1}{12}$ is determined by the death and birth rates as well as the inequality coefficient. These paremeters are chosen in order to satisfy (17), the necessary condition for attaining an equilibrium in an unequal society.

The same comments as in section 5.1.1 hold here if we choose a sub-optimal δ.

5.3.4 Unequal Society: Oscillatory Approach to Equilibrium

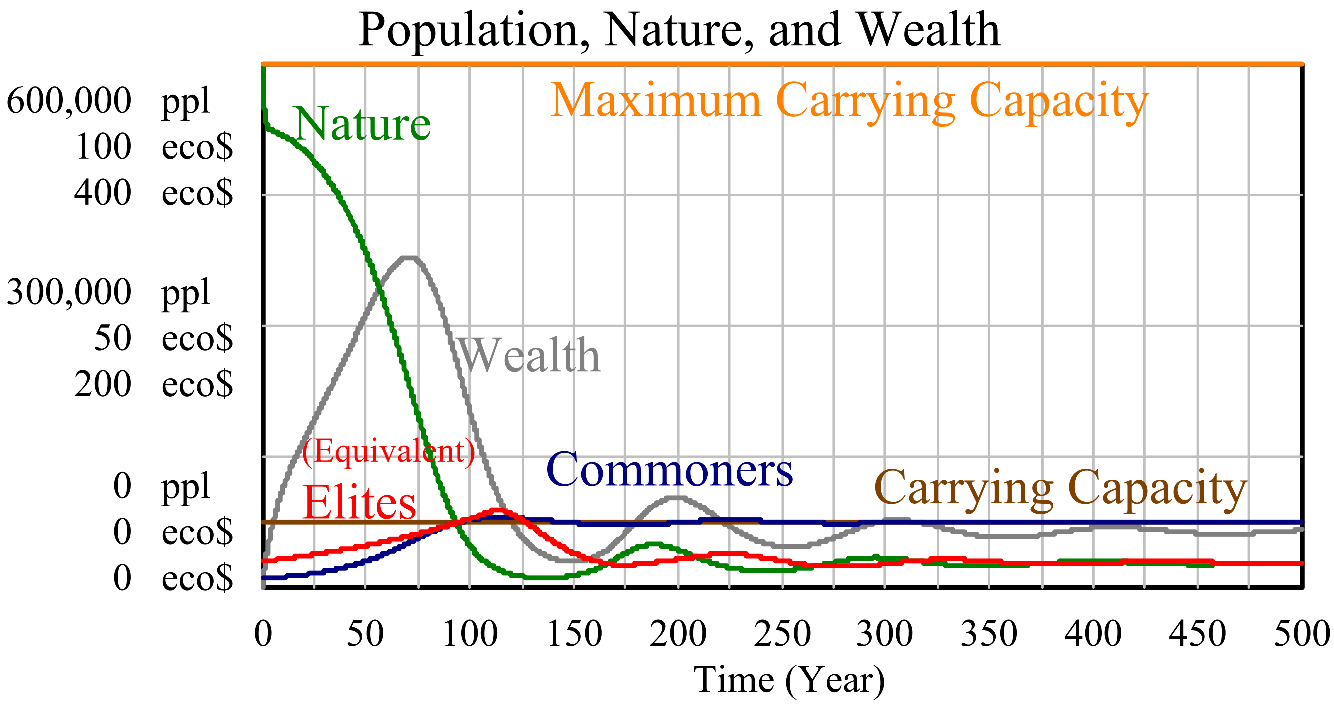

Figure 16: With $\delta \gtrsim \delta_{***}$, it is still possible to oscillate and converge to an equilibrium.

The parameter values and initial conditions in this scenario are exactly the same as the previous scenario, presented in section 5.3.3, except for δ. It is increased to 1.3×10^{-5}, almost $2\delta_{***}$. This results in a much lower Carrying Capacity compared to 5.3.3, as can be seen from a comparison of figures 15 and 16. Therefore, the total final population in the present scenario is much less than the total final population in the previous scenario, 5.3.3.

6 Discussion of Results

We conducted a series of experiments with the simple HANDY model, considering first an egalitarian society without Elites ($x_E = 0$), next an equitable society ($\kappa = 1$) where Non-Workers and Workers are equally paid, and finally an unequal society whose Elites consume κ times more than the Commoners. The model was also used to find a sustainable equilibrium value and the maximum carrying capacity within each of these three types of societies.

6.1 Unequal Society

The scenarios most closely reflecting the reality of our world today are found in the third group of experiments (see section 5.3), where we introduced economic stratification. Under such conditions, we find that collapse is difficult to avoid. Importantly, in the first of these unequal society scenarios, 5.3.1, even using an optimal depletion rate (δ_*) and starting with a very small number of Elites, *the solution appears to be on a sustainable path for quite a long time*, then Elites grow and consume too much, resulting in a famine among Commoners that eventually causes the collapse of society. This Type-I collapse is due to a loss of workers, rather than a collapse of Nature. Despite appearing initially to be the same as the sustainable optimal solution obtained in the absence of Elites, economic stratification changes the final result: the Elites become sizable and keep growing until the society collapses.

In scenario 5.3.2, with a larger depletion rate, the decline of the Commoners occurs faster, while the Elites are still thriving, but eventually the Commoners collapse completely, followed by the Elites. It is important to note that in both of these scenarios, the Elites —due to their wealth— do not suffer the detrimental effects of the environmental collapse until much later than the Commoners. We could posit that this buffer of wealth, as well as the initial apparently sustainable trajectory, allows Elites to continue "business as usual" despite the impending catastrophe. It is likely that this is an important mechanism that would help explain how historical collapses were allowed to occur by seemingly oblivious elites (most clearly apparent in the Roman and Mayan cases).

The final two scenarios in this set of experiments, 5.3.3 and 5.3.4, are designed to indicate the kinds of policies needed to avoid this catastrophic outcome. They show that, in the context of economic stratification, inequality must be greatly reduced and population growth must be strictly controlled in order to avoid a societal collapse [Daly, 2008].

6.2 Egalitarian Society

In order to further understand what conditions are needed to avoid collapse, our first set of experiments model a society without economic stratification and start with parameter values that make it possible to reach a maximum carrying capacity (scenario 5.1.1). The results show that in the absence of Elites, if the depletion per capita is kept at the optimal level of δ_*, the population grows smoothly and asymptotes the level of the maximum carrying capacity. This produces a soft-landing to equilibrium at the maximum sustainable population and production levels.

Increasing the depletion factor slightly (scenario 5.1.2) causes the system to oscillate, but still reach a sustainable equilibrium, although, importantly, at a lower carrying capacity. Population overshoots its carrying capacity, but since the overshoot is not by too much —of the order of the carrying capacity— the population experiences smaller collapses that can cause it to oscillate and eventually converge to a sustainable equilibrium. Thus, while social disruption and deaths would occur, a total collapse is avoided.

A further increase in the depletion factor (scenario 5.1.3) makes the system experience oscillatory periods of growth, very large overshoots and devastating collapses that almost wipe out society, but the eventual recovery of nature allows for the cycle to be repeated. These kinds of cycles of prosperous growth followed by overshoot and an almost complete collapse may be represented in the historical record

Increasing the depletion factor even further (scenario 5.1.4) results in a complete collapse of the system. This shows that depletion alone, if large enough, can result in a collapse —even in the absence of economic stratification.

6.3 Equitable Society (with Workers and Non-Workers)

As the second set of experiments (presented in section 5.2) show, HANDY allows us to model a diverse range of societal arrangements. In this set of experiments, choosing $x_E \geq 0$ and $\kappa = 1$ has allowed us to model a situation that can be described as having Workers and Non-Workers with the same level of consumption, i.e., with no economic stratification. The Non-Workers in these scenarios could represent a range of societal roles from students, retirees and disabled people, to intellectuals, managers, and other non-productive sectors. In this case, the Workers have to deplete enough of Nature to support both the Non-Workers and themselves.

The first scenario, 5.2.1, shows that even with a population of Non-Workers, the total population can still reach a sustainable equilibrium without a collapse. In scenario 5.2.2, we find that increasing the depletion factor induces a series of overshoots and small collapses where population eventually converges to a lower sustainable equilibrium. Like in an egalitarian society, scenario 5.2.3 shows us that increasing the depletion parameter further results in cycles of large overshooting, major collapses, and then eventual recovery of nature. Scenario 5.2.4 shows us that increasing depletion per capita further can produce a total collapse with no recovery.

Finally, scenario 5.2.5, which is a replication of 5.2.4 with a much higher ratio of Non-Workers to Workers, shows that a collapse in an equitable society could be avoided by reducing the average depletion per capita. We note that this scenario could also represent a situation where, rather than having paid Non-Workers, the workload per capita is reduced, with the whole population working "fewer days a week". Such a "work-sharing" policy has been successfully implemented in Germany over the past few years for reducing unemployment [Baker and Hasset, 2012; Hasset, 2009]. Moreover, Knight et al. [2012] show, through a panel analysis of data for 29 high-income OECD countries from 1970 to 2010, that reducing work hours can contribute to sustainability by reducing ecological strain. This conclusion agrees with our comparison of the two scenarios, 5.2.5 and 5.2.4, presented above.

6.4 HANDY and Brander-Taylor Model

As previously mentioned, a similar use of the predator-prey approach was applied in the pioneering work of Brander and Taylor [1998], hereafter called BT, to study the historical rise and fall of the Easter Island population. In comparison to their model, with just two equations for Population and Nature, the introduction of Elites and Commoners, and accumulated Wealth, results in a greater variety and broader spectrum of potential solutions. Moreover, the collapse scenario presented in BT is somewhat different from the ones presented above. As a matter of fact, the collapse scenario presented in figure 3 of BT seems to be more of an oscillatory approach to equilibrium, similar to the one shown in our figure 5, and not a collapse in the sense that we define in this paper. Furthermore, the carrying capacity, in the sense we define in this paper, is also different from what Brander and Taylor [1998] call carrying capacity. Indeed, their carrying capacity (K) is our maximum nature or Nature's capacity, λ.

Although our model development was carried out independently from what was done by Brander and Taylor, our underlying approach is the same. However, we make certain different assumptions, and develop a more complex model structure that can apply to several types of societies with different socioeconomic structures. Unlike works that tend to study further implications of the two-dimensional model of BT [Anderies, 2000], the model we have developed introduces a more complex set of possible feedbacks and non-linear dynamics, and a greater spectrum of potential outcomes than the model presented in BT. This allows HANDY to model a different and wider set of thought experiments.

7 Summary and Future Work

Collapses of even advanced civilizations have occurred many times in the past five thousand years, and they were frequently followed by centuries of population and cultural decline and economic regression. Although many different causes have been offered to explain individual collapses, it is still necessary to develop a more general explanation. In this paper we attempt to build a simple mathematical model to explore the essential dynamics of interaction between population and natural resources. It allows for the two features that seem to appear across societies that have collapsed: the stretching of resources due to the strain placed on the ecological carrying capacity, and the division of society into Elites (rich) and Commoners (poor).

The Human And Nature DYnamical model (HANDY) was inspired by the Predator and Prey model, with the human population acting as predator and nature being the prey. When small, Nature grows exponentially with a regeneration coefficient γ, but it saturates at a maximum value λ. As a result, the maximum regeneration of nature takes place at $\lambda/2$, not at the saturation level λ. The Commoners produce wealth at a per capita depletion rate δ, and the depletion is also proportional to the amount of nature available. This production is saved as accumulated wealth, which is used by the Elites to pay the Commoners a subsistence salary, s, and pay themselves κs, where κ is the inequality coefficient. The population of Elites and Commoners grow with a birth rate β and die with a death rate α which remains at a healthy low level when there is enough accumulated food (wealth). However, when the population increases and the wealth declines, the death rate increases up to a famine level, leading to population collapse.

We show how the carrying capacity —the population that can be indefinitely supported by a given environment [Catton, 1980]— can be defined within HANDY, as the population whose total consumption is at a level that equals what nature can regenerate. Since the regrowth of Nature is maximum when $y = \lambda/2$, we can find the optimal level of depletion (production) per capita, δ_* in an egalitarian society where $x_E \equiv 0$, $\delta_{**}(\geq \delta_*)$ in an equitable society where $\kappa \equiv 1$, and δ_{***} in an unequal society where $x_E \geq 0$ and $\kappa \geq 1$.

In sum, results of our experiments, discussed in section 6, indicate that either one of the two features apparent in historical societal collapses —over-exploitation of natural resources and strong economic stratification— can independently result in a complete collapse. Given economic stratification, collapse is very difficult to avoid and requires major policy changes, including major reductions in inequality and population growth rates. Even in the absence of economic stratification, collapse can still occur if depletion per capita is too high. However, collapse can be avoided and population can reach equilibrium if the per capita rate of depletion of nature is reduced to a sustainable level, and if resources are distributed in a reasonably equitable fashion.

This version of HANDY so far contains only one region, and only renewable natural resources. In the next version, we plan to include several extensions including:

- Disaggregation of Nature into nonrenewable stocks, renewable stocks, and flows.
- The introduction of "government policies" that can modify parameters such as depletion, the coefficient of inequality and birth rate, to see whether it is possible to avoid a collapse when the carrying capacity is exceeded.
- The introduction of multiple coupled regions to represent countries with different policies, trade carrying capacity and resource wars.

We have posted HANDY on `http://www.atmos.umd.edu/~ekalnay/handy-ver1.mdl` We welcome our readers to download the code, perform other experiments, and post their results at the same webpage.

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